XIII Colóquio Brasileiro de Ciências Geodésicas • 2024
Universidade Federal do Paraná
25 Anos
Conectando mentes e provendo conhecimento

DESIGNING GEODETIC MONITORING NETWORKS: ASSESSING MINIMAL DETECTABLE DISPLACEMENT (MDD) WITH CONFIDENCE AND SENSITIVITY ANALYSIS

Felipe Carvajal Rodríguez [1,3], Ivandro Klein [2,3], Luís Koenig Veiga [3]

[1,3] Universidad Bernardo O'Higgins, Ingeniería en Geomensura y Cartografía Advance, Escuela Ciencias de la Tierra, Santiago, Chile– felipecarvajal@docente.ubo.cl
[2] Dept. of Civil Construction, Federal Institute of Santa Catarina, Florianopolis City, SC 88020-300, Brazil– ivandroklein@gmail.com
[3] Graduate Program in Geodetic Sciences (PPGCG) at Federal University of Paraná (UFPR), Curitiba City, PR,81.531-990, Brazil– kngveiga@gmail.com

Curitiba, 26 a 29 de novembro de 2024

Introduction

- Accuracy and sensitivity criteria are commonly used in the pre-analysis of geodetic monitoring networks. While the accuracy ensures positional quality, the sensitivity analysis measures the network's ability to detect displacements.

- According to [1] both approaches can be presented as ellipsoids. The axes of these ellipsoids can be computed through Principal Component Analysis (PCA) and Minimal Detectable Displacements (MDD) [2,3,4]. The relationship between both ellipsoids only concerns to a size comparison due to a lack of theoretical basis.

- This condition also applies to the significance tests of computed displacements, where sensitivity characteristics are not considered. Thus, [1] provide a theoretical foundation that uniquely joint sensitivity with confidence and significance analysis.

Introduction

- The approach outlined by [1] determines the sizes of significance, sensitivity, and confidence ellipsoids using MDD values and considers the inequality that involve the critical displacement values $u_{h,0}$ and the noncentrality parameter $\lambda_{h,\alpha_0,\beta_0}$.
- In particular, this approach coordinates the h −dimensional displacement vector with probability levels, including the significance level or Type I error rate (α_0), the false negative or Type II error rate (β_0), and the test's power ($\gamma_0 = 1 - \beta_0$).
- The main difference between accuracy (significance or confidence) and sensitivity ellipsoids is that the former disregards the probability of false negatives (β_0). Thus, for a specific value Type II error rate (β_0), called coordinate value of Type II error ($\underline{\beta_0}$) which depends of the h −dimensional displacement vector and α_0, the inequality mentioned above becomes the equality $u_{h,0} = \lambda_{h,\alpha_0,\underline{\beta_0}}$.

Introduction

- Here, setting $\alpha_0 = 1 - CL$ (where CL denotes the confidence level) reduces the analysis to two ellipsoids: confidence and sensitivity. Therefore, under this approach the MDD measures depends on variables related to the statistic test, such as h, α_0, β_0.

- However, the research of [1] does not analyze aspects such as the network spatial dimension (1D, 2D, 3D), functional model, or their role in the pre-analysis or design of geodetic monitoring networks.

- Thus, this work presents an analysis of these aspects under the unified approach of [1], focusing on the design of geodetic monitoring networks and extending the findings of [5].

Introduction

Three experiments are presented:

- An analysis of the spatial dimension of the network,
- The influence of functional model and;
- An example of GNSS (Global Navigation Satellite System) monitoring network design.

Experiments

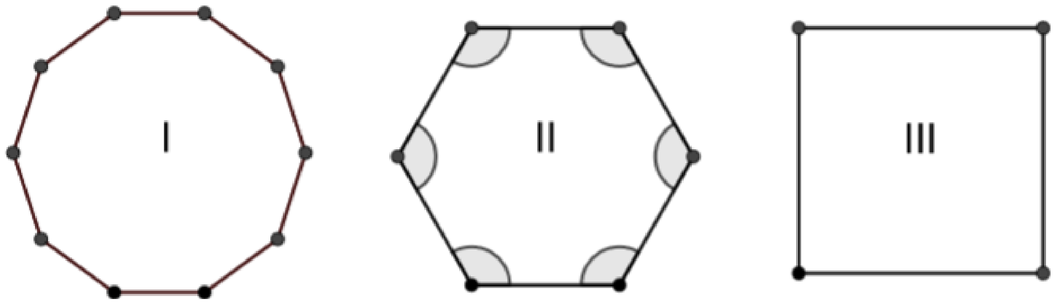

Figure 1 – equal h –dimensional networks ($h = rank(C_{\hat{d}}) = 9$): I correspond to a 1D leveling network, II corresponds to a 2D horizontal network and III corresponds to a 3D GNSS network.

Table 1 – MDD values (mm) for geodetic networks (global analysis with $\alpha_0 = 5\%$ and $h = rank(C_{\hat{d}}) = 9$).

Spatial dimension	**Sensitivity ($\beta_0 = 4\%$)**		**Confidence ($\beta_0 = 16\%$)**		**Sensitivity ($\beta_0 = 20\%$)**	
	MIN	**MAX**	**MIN**	**MAX**	**MIN**	**MAX**
1D	4.4	14.4	2.9	9.4	2.8	9.1
2D	2.5	8.9	1.7	5.8	1.6	5.6
3D	4.4	6.3	2.9	4.1	2.8	4.0

Source: Author (2024).

Experiments

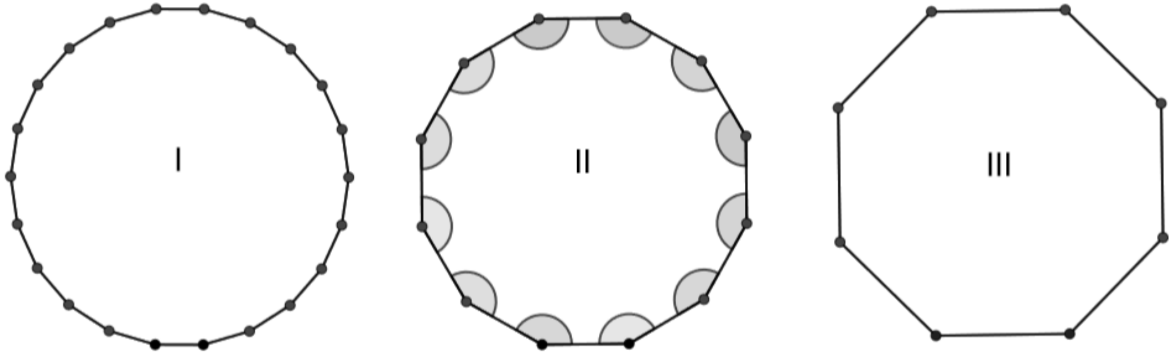

Figure 2 – equal h −dimensional networks ($h = rank(C_{\hat{d}}) = 21$): I correspond to a 1D leveling network, II corresponds to a 2D horizontal network and III corresponds to a 3D GNSS network.

Table 2 – MDD values (mm) for geodetic networks (global analysis with $\alpha_0 = 5\%$ and $h = rank(C_{\hat{d}}) = 21$).

Spatial dimension	**Confidence ($\beta_0 = 4\%$)**		**Sensitivity ($\beta_0 = 16\%$)**		**Sensitivity ($\beta_0 = 20\%$)**	
	MIN	**MAX**	**MIN**	**MAX**	**MIN**	**MAX**
1D	4.0	28.4	3.6	24.9	3.3	23.0
2D	2.1	15.6	1.8	13.6	1.7	12.6
3D	4.0	10.6	3.6	9.2	3.3	8.5

Source: Author (2024).

Experiments

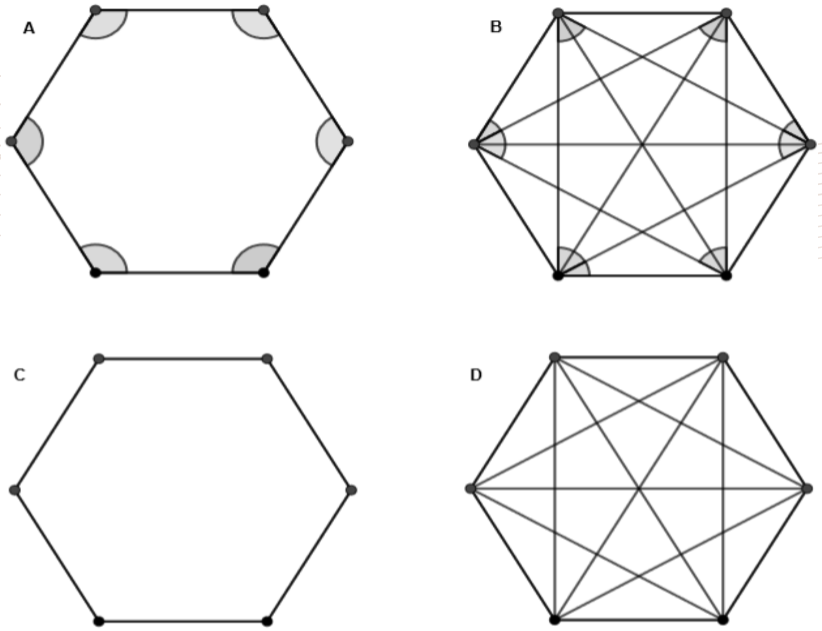

Figure 3 – A and B correspond to a 2D network of "Case I" with low and high redundancy (respectively), C and D correspond to a 2D network of "Case II" with low and high redundancy (respectively), $h = rank(C_{\hat{d}}) = 9$ in all cases.

Table 3 – MDD values (mm) for 2D horizontal networks (global analysis with $\alpha_0 = 5\%$, $\beta_0 = 20\%$, and $h = rank(C_{\hat{d}}) = 9$).

Approach	**Case I**		**Case II**	
	MIN	**MAX**	**MIN**	**MAX**
Low redundancy	1.6	5.6	2.8	5.6
High redundancy	1.3	3.0	2.3	2.3

Source: Author (2024).

Experiments

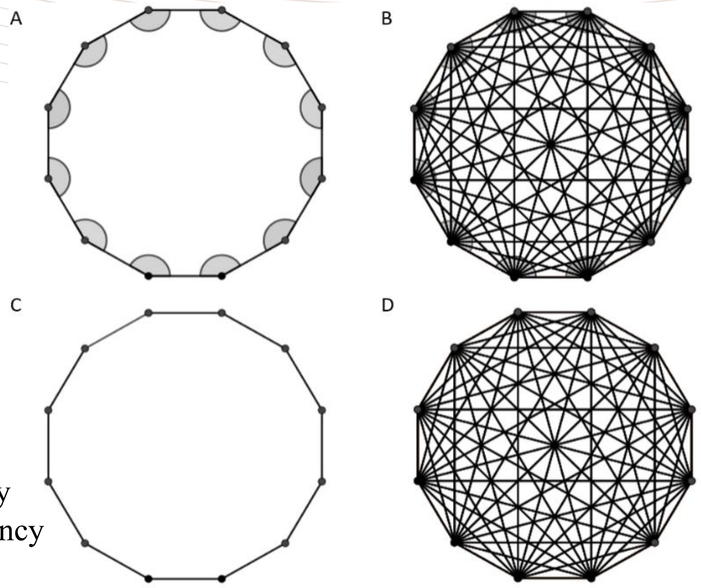

Figure 4 – A and B correspond to a 2D network of "Case I" with low and high redundancy (respectively), C and D correspond to a 2D network of "Case II" with low and high redundancy (respectively), $h = rank(C_{\hat{d}}) = 21$ in all cases.

Table 4 – MDD values (mm) for 2D horizontal networks (global analysis with $\alpha_0 = 5\%$, $\beta_0 = 20\%$, and $h = rank(C_{\hat{d}}) = 21$).

Approach	**Case I**		**Case II**	
	MIN	**MAX**	**MIN**	**MAX**
Low redundancy	1.7	12.6	3.3	12.6
High redundancy	1.0	2.6	1.9	1.9

Source: Author (2024).

Experiments

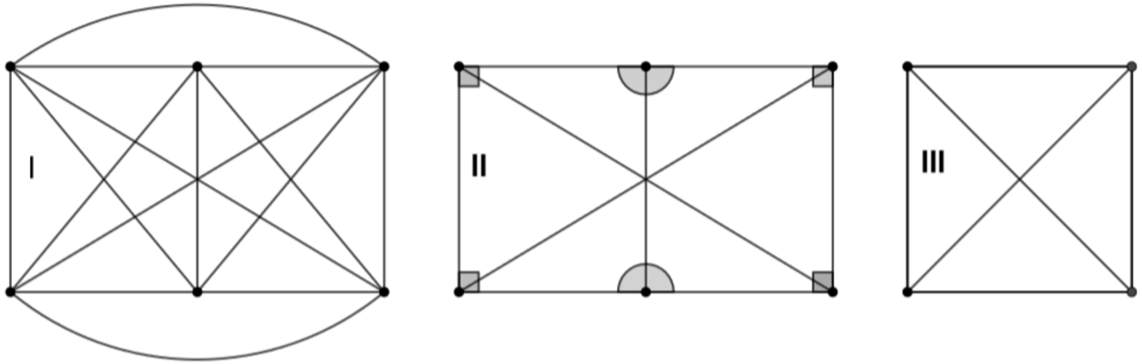

Figure 5 – I correspond to a 2D trilateration network, II corresponds to a 2D triangulateration network and III corresponds to a 3D GNSS network, $h = rank(C_{\hat{a}}) = 9$ in all cases.

Table 5 – MDD values (mm) for geodetic networks of Figure 5 (global analysis with $\alpha_0 = 5\%$ and $h = rank(C_{\hat{a}}) = 9$)

Approach	Trilateration 2D		Triangulateration 2D		GNSS 3D	
	MIN	**MAX**	**MIN**	**MAX**	**MIN**	**MAX**
Confidence ($\beta_0 = 16\%$)	2.5	5.6	1.8	4.4	2.9	2.9
Sensitivity ($\beta = 20\%$)	2.4	5.4	1.7	4.2	2.8	2.8

Source: Author (2024).

Experiments

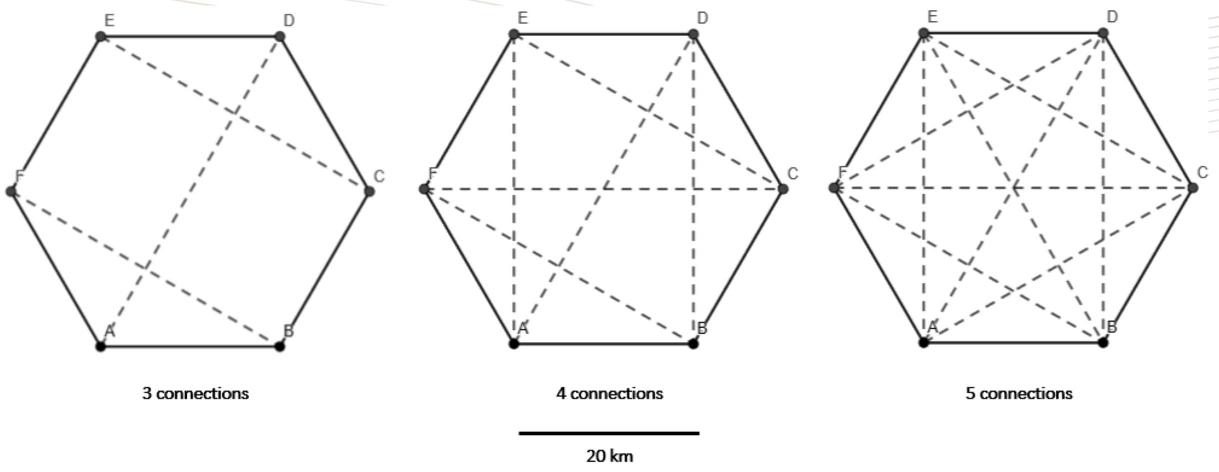

Figure 6 – GNSS monitoring network with 3, 4 and 5 connections per point.

Table 6 – Maximum MDD values (mm) for GNSS monitoring network (local analysis with $\alpha_0 = 5\%$ and $h = rank(C_{\hat{d}}) = 3$)

Connections Per point	Time session: 1 hour			Time session: 2 hours		
	Confidence ($\underline{\beta}_0 = 36\%$)	**Sensitivity ($\beta = 20\%$)**	**Sensitivity ($\beta = 5\%$)**	**Confidence ($\underline{\beta}_0 = 36\%$)**	**Sensitivity ($\beta = 20\%$)**	**Sensitivity ($\beta = 5\%$)**
3	24.0	28.4	35.6	16.8	19.9	24.9
4	20.0	23.7	29.7	14.0	16.5	20.8
5	17.6	20.8	26.1	12.3	14.5	18.2

Source: Author (2024).

Table 7 – Optimal solutions for the GNSS monitoring network with a threshold of 20 mm for MDD values (local analysis with $\alpha_0 = 5\%$ and $h = rank(C_{\hat{d}}) = 3$)

Confidence ($\underline{\beta}_0 = 36\%$)	**Sensitivity ($\beta = 20\%$)**	**Sensitivity ($\beta = 5\%$)**
5 connections per point and 3 sessions with 1 hour each (3 hours in total)	3 connections per point and 2 sessions with 2 hours each (4 hours in total)	5 connections per point and 3 sessions with 2 hours each (6 hours in total)

Source: Author (2024).

Conclusions

The main conclusions of this work are as follows:

- Larger network dimensions result in smaller max MDD values for a given $h = rank(C_{\hat{d}})$.
- The differences in max MDD values between networks of different dimensions increase with *h* in both confidence and sensitivity analyses.
- Increasing redundancy is more efficient for 1D networks, which typically have higher max MDD values than 3D networks.
- the practical importance of considering both absolute MDD values and false negative probability in the pre-analysis of geodetic monitoring networks.

References

[1] PRÓSZYŃSKI, W.; ŁAPIŃSKI, S. Investigating support by minimal detectable displacement in confidence region determination and significance test of displacements. **Journal of Geodesy**, 95(10), 112 2021.

[2] PELZER, H. Nachweis der Staumauerdeformationen unter Anwendung statistischer Verfahren. **Berichte Arbeitsgruppe B**, Deutscher Geodätentag Braunschweig, 1972.

[3] NIEMEIER, W. Principal component analysis and geodetic networks - some basic considerations. **Proc. Survey Control Networks**, 7, 275–291, 1982.

[4] HSU, R.; HSIAO, K. Pre-computing the sensitivity of a GPS station for crustal deformation monitoring. **Journal of the Chinese Institute of Engineers**, 25(6), 715–722. 2002

[5] CARVAJAL, F., KLEIN, I., DE SOUZA OLIVEIRA ALVES, S., AND KOENING VEIGA, L. Influence of network configuration and stochastic model on the determination of the minimum detectable displacements (MDD) through sensitivity analysis and significance test. **Bulletin of Geodetic Sciences**, 30(1), 2024.

[6] OGUNDARE, J. **Understanding Least Squares Estimation and Geomatics Data Analysis**. Wiley, 10.1002/9781119501459, 2018

[7] KOCH, I., KLEIN, I., GONZAGA DA SILVEIRA JR, L., ROFATTO, V., MATSUOKA, M., MONICO, G., AND VERONEZ M. Metaheuristic-based stochastic models for GNSS relative positioning planning. **GPS Solutions**, 28, 1. 2023.

AGRADECIMENTOS

GEENG
Laboratório de Geodésia Aplicada à Engenharia

PÓS-GRADUAÇÃO EM CIÊNCIAS GEODÉSICAS

UNIVERSIDAD BERNARDO O'HIGGINS
SERIEDAD Y CALIDAD